ference, to determine the average densities in the stagnant zone to within 15%. The quality of the numerical solution is improved when the bow shock wave is isolated or the grid is made finer in the shock layer (in the calculation of 10 cells in the shock layer). A modification of the calculating model is required to increase the accuracy of the solution in the separation zone: The Reynolds equations must be used jointly with any of the multiparametric models of turbulence in the entire separation zone.

NOTATION

d, disk diameter; D, body diameter; ℓ, disk extension; δ, thickness of the shear layer; x^*, coordinate along the shear layer; ν_t, turbulent viscosity; c_n, c_t, empirical constants; C_x, C_{xb}, C_{xw}, coefficients of total aerodynamic, base, and wave resistance.

LITERATURE CITED

1. I. A. Belov, Interaction of Inhomogeneous Streams with Obstacles [in Russian], Mashinostroenie, Leningrad (1983).
2. P. I. Kovalev, N. P. Mende, A. N. Mikhalev, et al., "Use of a Mach—Zehnder interferometer to study moving objects," in: Optical Methods of Research in Ballistic Experiments [in Russian], Nauka, Leningrad (1979), pp. 70-90.
3. V. A. Komissaruk and N. P. Mende, "Experience in using diffraction and polarization interferometers in ballistic experiments," in: Optical Methods of Research in Ballistic Experiments [in Russian], Nauka, Leningrad (1979), pp. 91-113.
4. I. A. Vereninov, I. M. Dement'ev, and A. N. Mikhalev, "Tomography of the field of separation flow over a freely flying body," in: Abstracts of Papers of the First Symposium on Computer Tomography [in Russian], Vychisl. Tsent., Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1983), pp. 36-37.
5. V. G. Ivanov, "A comparative analysis of the possibilities of optical methods in the investigation of turbulence," in: Optical Methods of Research in Ballistic Experiments [in Russian], Nauka, Leningrad (1979), pp. 200-215.
6. S. K. Godunov (ed.), Numerical Solution of Multidimensional Problems of Gasdynamics [in Russian], Nauka, Moscow (1976).
7. W. C. Reynolds and T. Cebeci, "Calculation of turbulent flows," in: Turbulence, P. Bradshaw (ed.), 2nd edn., Springer-Verlag, Berlin—New York (1978), pp. 193-229.

INFLUENCE OF THERMAL RADIATION ON THE STRUCTURE OF THE TEMPERATURE FIELD IN A TURBULENT FLOW

V. P. Kabashnikov and A. A. Kurskov

UDC 536.24:532.517.4

The form of the structural function for temperature fluctuations in the turbulent flow of a radiation gas is set up theoretically.

An important characteristic of the temperature field in a turbulent flow is the structural function for the temperature [1], which is a dependence of the root-mean-square of the temperature difference at two points on the distance between them. The structural function characterizes the amplitude of the temperature fluctuations of different spatial scales and, therefore, also the microstructure of the temperature field.

The form of the structural function for nonradiation media was first set up in [2, 3]. The influence of radiation on turbulent temperature fluctuations was investigated in [4-8]. Paper [7], where conditions were indicated for which the times of radiative decay of the temperature perturbations of different scales are comparable to the times of hydrodynamic decay,

---

Institute of Physics, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 5, pp. 773-780, November, 1985. Original article submitted October 29, 1984.

0022-0841/85/4905-1299$09.50

is closest in subject matter to the present paper. In substance, these are conditions for which radiation starts to influence the structural function. However, the very form of the structural function in a radiating medium has apparently not been examined up to now.

The temperature field in a statistically homogeneous infinite gas medium with thermal radiation taken into account is described by the equation

$$\frac{\partial T(\mathbf{x}, t)}{\partial t} + \mathbf{v}(\mathbf{x}, t)\nabla T(\mathbf{x}, t) = \chi \Delta T(\mathbf{x}, t) + q(\mathbf{x}, t) - 4\pi(\rho c_p)^{-1}\int_0^\infty d\omega \varkappa_\omega \int dz W(z)[B_\omega(\mathbf{x}, t) - B_\omega(\mathbf{x}-\mathbf{z}, t)], \quad (1)$$

where $T(\mathbf{x}, t)$ and $\mathbf{v}(\mathbf{x}, t)$ are, respectively, the gas temperature and velocity at a point $\mathbf{x}$ at a time t; B_ω is the Planck function; ω is the radiation frequency; κ_ω is the absorption coefficient; $q(\mathbf{x}, t)$ is the power of random heat sources; and

$$W(z) = (4\pi z^2)^{-1}\varkappa_\omega \exp(-\varkappa_\omega z). \quad (2)$$

For simplicity, we examine a "gray" gas ($\kappa_\omega = \kappa$) and we assume that ρ and χ are constants.

The gas velocity and the rate of heat liberation in (1) are random functions. Consequently, the temperature, which is a functional of the velocities of the medium and the heat liberation, also becomes random.

To obtain an equation for the structural function, (1) must be multiplied by the temperature at adjacent points and the average must be taken. The main difficulty that occurs here is the evaluation of the correlation between the velocity of the medium and the product of the temperatures as well as of the temperature with the rate of heat liberation. To overcome it we assume that the velocities of the medium and the heat liberation are Gaussian δ-correlated processes in time. This permits utilization of the method of calculating the mean product of a function and a functional [9], for which a detailed exposition is contained in [10, 11], to determine the correlations mentioned. Let the velocities of the medium and the heat liberation be statistically stationary, homogeneous, isotropic, and not mutually correlated random functions with zero means. Then taking account of the incompressibility of the gas as well as the temperature fluctuations that permit linearization of the integrand in (1), we obtain the following equation for the structure function of a temperature field $D(x) = \langle [T(\mathbf{x}+\mathbf{y}, t) - T(\mathbf{y}, t)]^2 \rangle$:

$$x^{-2}\{x^2[R(x) + 2\chi]D'\}' - hD + h\int W(y) D(|\mathbf{x}+\mathbf{y}|)d\mathbf{y} = 4N(x), \quad (3)$$

where $R(x) = x_i x_j x^{-2} R_{ij}$ is the longitudinal part of the structural tensor R_{ij}:

$$R_{ij}(x) = \int_{-\infty}^{t} \langle [v_i(\mathbf{y}, t) - v_i(\mathbf{y}+\mathbf{x}, t)][v_j(\mathbf{y}, t_1) - v_j(\mathbf{y}+\mathbf{x}, t_1)] \rangle \, dt_1. \quad (4)$$

The quantity h has the meaning of a radiation cooling rate for an optically thin gas volume

$$h = \frac{32\varkappa\sigma \langle T \rangle^4}{\rho c_p \langle T \rangle}, \quad (5)$$

where σ is the Stefan–Boltzmann constant, and $\langle T \rangle$ is the mean absolute gas temperature

$$N(x) = \int_{-\infty}^{t} \langle q(\mathbf{y}, t) q(\mathbf{x}+\mathbf{y}, t_1) \rangle dt_1. \quad (6)$$

The function R(x) with the meaning of the coefficient of turbulent thermal diffusivity can be estimated as follows:

$$R(x) \sim \begin{cases} \nu\eta^{-2}x^2, & x \ll \eta, \quad (7) \\ \nu\eta^{-4/3}x^{4/3} \sim \varepsilon^{1/3}x^{4/3}, & \eta \ll x \ll L, \quad (7a) \\ \varepsilon^{1/3}L^{4/3}, & x \gg L. \quad (7b) \end{cases}$$

The characteristic scale of the source function N(x) is taken equal to the external scale of the velocity field L by assuming that $N(x) \cong N(0) = N$ for values of x belonging to the dissipation interval and inertial interval.

Let us turn to an analysis of the fundamental equation (3). If there is no radiation ($h = 0$), then (3) is solved exactly

$$D(x) = \int_0^x dyy^{-2}[R(y) + 2\chi]^{-1} \int_0^y 4N(z) z^2 dz. \tag{8}$$

For the majority of substances $\nu \sim \chi$. Using this condition as well as the dependences (7), we obtain

$$D(x) \sim \begin{cases} N(3\chi)^{-1}x^2, & x \ll \eta, & (9) \\ 2N\varepsilon^{-1/3}x^{2/3}, & \eta \ll x \ll L, & (9a) \\ 2N\varepsilon^{-1/3}L^{2/3}, & x \gg L, & (9b) \end{cases}$$

which agrees with the results in [2, 3]. Qualitatively, the correct form of the structural function that follows from (3) in the absence of radiation permits the hope that application of (3) to investigate the influence of radiant heat transfer will also result in physically reasonable results.

Let us examine the influence of radiation on the structural function.

1. Let the path length of a radiation quantum ℓ, defined as the reciprocal absorption coefficient, be less than the internal scale of turbulence ($\ell \ll \eta$). To clarify the form of the structural function in the dissipation interval, we discard the coefficient of turbulent thermal diffusivity $R(x)$, which is small compared with the quantity χ, and we solve the differential equation obtained by considering the integral term a known function. We consequently obtain

$$D(x) = \frac{h}{12\chi} F(0) x^2, \quad x \ll \eta, \tag{10}$$

where

$$F(x) = \frac{4N(x)}{h} - \int W(y) D(|x + y|) dy. \tag{11}$$

The values of the arguments of the structural function under the integral sign in (11) are bounded by the quantum path length $\ell \ll \eta$ for $x = 0$, which permits the determination of $F(0)$ by using (10) and finally obtaining

$$D(x) = 2Nx^2(6\chi + hl^2)^{-1}. \tag{12}$$

For $x \gg \ell$, and particularly for $x \gg \eta$, the structural function changes slightly in an interval of order ℓ, which permits utilization of the radiant heat conduction approximation:

$$x^{-2}\left\{x^2\left[R(x) + 2\chi + \frac{1}{3}hl^2\right]D'\right\}' = 4N(x). \tag{13}$$

The solution of (13) differs from the solution of (8) in that the thermal diffusivity coefficient in (8) is replaced by the sum of molecular and radiant thermal diffusivity coefficients:

$$D(x) = \int_0^x dyy^{-2}\left[R(y) + 2\chi + \frac{1}{3}hl^2\right]^{-1} \int_0^y 4N(z) z^2 dz. \tag{14}$$

It follows from (12)-(14) that the minimal intensity at which radiation influences the form of the structural function is determined by the condition

$$hl^2 \sim \chi. \tag{15}$$

The action of the radiation appears primarily in the dissipation interval, resulting in a diminution in the steepness of the quadratic section of the structural function as compared with the case of a nonradiating gas. As the radiation intensity grows, the x^2 section is propagated into the inertial domain of scales, as seen from (14), to the value of the argument x_1 defined by the relationship

$$R(x_1) \sim hl^2. \tag{16}$$

For $x < x_1$, the structural function is independent of convective heat transfer and is determined completely by radiation transport and molecular heat conduction. For $x > x_1$, the influence of radiation on the form of the structural function is negligible.

If the radiation intensity is so large that

$$hl^2 \gg R(L), \tag{17}$$

then (16), which defines the boundary of the section perturbed by radiation, has no meaning. In this case we obtain from (14)

$$D(x) = 12(hl^2)^{-1}\left\{\int_0^x N(z)\,zdz - x^{-1}\int_0^x N(z)\,z^2dz\right\}, \tag{18}$$

from which it follows that $D = [2N(0)/h\ell^2]x^2$ for $x \ll L$, where $N(x)$ can be considered constant, and depends substantially on the form of the source function $N(x)$ for $x \sim L$. It is also seen from (18) that further growth of the radiation intensity results in a uniform diminution in the structural function for all values of its argument.

Some of the results obtained can be interpreted by starting from relationships between the radiation τ_x^r and hydrodynamic τ_x decay times for a temperature inhomogeneity of dimension x:

$$\tau_x^r \sim \begin{cases} h^{-1}, & x \ll l, \\ h^{-1}l^{-2}x^2, & x \gg l, \end{cases} \tag{19}$$

$$\tau_x \sim \begin{cases} \chi^{-1}x^2, & x \ll \eta, \\ \varepsilon^{-1/3}x^{2/3}, & \eta \ll x \ll L. \end{cases} \tag{20}$$

If the radiation time is much greater than the hydrodynamic time, then the influence of radiation is not essential. The condition for the beginning of influence (15) corresponds to equality of the times (19) and (20) for $\ell \leqq x \leqq \eta$. As the radiation intensity grows further (increase in h), the domain of scales where radiation decay of the inhomogeneity proceeds more rapidly than the hydrodynamic, i.e., the domain of strong influence of radiation, expands. Its boundary x_1, determined by the equality of the times (19) and (20), agrees with that obtained from (16).

2. Let the quantum path length be in the inertial interval of scales ($\eta \ll \ell \ll L$). The minimal radiation intensity influencing the form of the structural function is determined by the condition

$$h^{-1} \sim \tau_l, \tag{21}$$

which can be obtained by using perturbation theory by utilizing the structural function undisturbed by radiation in (3). However, the case of strong radiation characterized by the inequality $h^{-1} \ll \tau_\ell$ is of fundamental interest.

The behavior of the structural function in the inertial scale interval can be clarified by substituting the turbulent thermal diffusivity coefficient (7a) into (3) and neglecting the quantity $\chi \ll R(x)$. Considering the function F(x) defined by the relationship (11) to be known, as before, we obtain

$$D(z) = -F(z) - z^{-7/2}K_{7/2}(z)\int_0^z y^{-7/2}I_{7/2}(y)(y^8F')'dy - z^{-7/2}I_{7/2}(z)\int_z^\infty y^{-7/2}K_{7/2}(y)(y^8F')'dy, \tag{22}$$

where $I_{7/2}$ and $K_{7/2}$ are Bessel function sof imaginary argument of order $^7/_2$

$$z = (x/x_0)^{1/3}; \tag{23}$$

$$x_0 = \frac{1}{27}\varepsilon^{1/2}h^{-3/2}; \tag{24}$$

x_0 is the scale at which the hydrodynamic decay time is compared to the volume deexcitation time h^-. It is easy to see that $x_0 << \ell$ in the case of strong radiation.

For small and large z the function D(z) has simple asymptotics:

$$D(z) = \frac{F(0)}{18} z^2, \quad z \ll 1, \tag{25}$$

$$D(z) = -F(z) - \frac{8}{z} F'(z) - F''(z), \ z \gg 1, \tag{26}$$

where the prime denotes differentiation with respect to z.

Expanding the structural function $D(|x + y|)$ in (11) into power series in x and y up to quadratic terms inclusively for

$$x_0 \ll x \ll l \tag{27}$$

and

$$x \gg l \tag{28}$$

repsectively, and substituting the expressions obtained for F(z) into (26), it can be shown that in the domain of values

$$x_0 \ll x \ll x_1, \tag{29}$$

where x_1 is determined from equality of the hydrodynamic and radiation decay times in the radiant heat conduction mode

$$\varepsilon^{-1/3} x_1^{2/3} = h^{-1} x_1^2 l^{-2}, \tag{30}$$

the products in (26) can be neglected and the following equation obtained:

$$D(x) = \int W(y) D(|\mathrm{x} + \mathrm{y}|) dy - 4h^{-1}N, \tag{31}$$

which does not contain hydrodynamic terms whose solution for constant N has the form

$$D(x) = 2N(hl^2)^{-1}x^2. \tag{32}$$

From the merger condition for the solutions (32) and (26) we obtain the value of the constant F(0) for x_0. Hence, the structural function determined by (25) and (10) for $x < x_0$ acquires the form of an unperturbed structural function with power of the fluctuation sources diminished $(h\tau_\ell)^3$ times:

$$D(x) \sim \begin{cases} N(h\tau_l)^{-3}(3\chi)^{-1}x^2, & x \ll \eta, \\ 2N(h\tau_l)^{-3}\varepsilon^{-1/3}x^{2/3}, & \eta \ll x \ll x_0. \end{cases} \tag{33}$$

Behavior of the structural function in the domain $x > x_1$ can be investigated on the basis of the asymptotic relationship (26); however, because of the inequality $x_1 >> \ell$, it is more natural to use the radiant heat conduction approximation that yields the same result as (26) in the inertial range but is also applicable for values of the arguments commensurate with the magnitude of the external scale of turbulence. In the mentioned approximation we have for the inertial interval for $x >> \ell$

$$D(x) \sim \int_0^x 4Nydy\, [3\varepsilon^{1/3} y^{4/3} + hl^2]^{-1}. \tag{34}$$

In the domain $x < x_1$, the expression (34) agrees with (32), which is not surprising since the domains of applicability of these expressions overlap for $\ell < x < x_1$. In the domain $x > x_1$, as seen from (34), (14), and (8), the structural function is exactly the same as in a nonradiating gas.

It was assumed implicitly in the preceding discussions that $x_1 << L$. If the mentioned inequality is spoiled, as holds under the condition

$$h \gtrsim \tau_l^{-1} L^{4/3} l^{-4/3}, \tag{35}$$

then the structural function is determined by (18) for x > ℓ.

3. Let the quantum path length exceed the external scale of turbulents (ℓ >> L). In this case, the integral term in (3) equals D(∞) and takes the form

$$x^{-2}\{x^2[R(x)+2\chi]D'\}' - h[D - D(\infty)] = 4N(x). \tag{36}$$

It can be shown by using perturbation theory that radiation starts to influence the form of the structural function noticeably for

$$h^{-1} \sim \tau_L. \tag{37}$$

Estimating the order of magnitude of terms in (36) associated with convection and heat conduction by means of the relationship

$$x^{-2}\{x^2[R(x)+2\chi]D'\}' \sim D(x)\tau_x^{-1} \tag{38}$$

and comparing (38) with terms taking account of the influence of radiation, we can conclude that for sufficiently high intensities two scale domains exist with an opposite relationship between the radiation and hydrodynamic types separated by the scale x_0 at which the times mentioned are in agreement. We limit ourselves to the case when x_0 belongs to the inertial interval of the scales.

In the domain $x > x_0$, where radiation predominates over hydrodynamics, (36) is valid without the convection—heat conduction terms. Its solution has the form

$$D(x) = 4h^{-1}[N(0) - N(x)]. \tag{39}$$

Let us note that the result (39) also follows from (26) in the inertial range.

The relationships (10) and (25) are valid in the domain $x < x_0$. To estimate F(0) we combine the solutions (39) and (25) at the point x_0 by assuming that for x << L

$$N(x) \sim N(0)[1 - x^2 L^{-2}]. \tag{40}$$

Consequently, we obtain the structural function for $x < x_0$

$$D(x) \sim \begin{cases} N(h\tau_L)^{-3}(3\chi)^{-1}x^2, & x \ll \eta, \\ 2N(h\tau_L)^{-3}\varepsilon^{-1/3}x^{2/3}, & \eta \ll x \ll x_0. \end{cases} \tag{41}$$

It has the same dependence on the distance as in the absence of radiation; however the fluctuation amplitude is diminished $(h\tau_L)^{3/2}$ times.

4. Therefore, the influence of radiation on the form of the structural function reduces qualitatively to the following.

For ℓ << η, radiation heat transfer is felt primarily in the dissipation interval where the structural function becomes more and more shallow as the radiation grows when conserving the x^2 dependence. The boundary of the square section is here shifted to the inertial scale interval. Outside the square section the structural function is the same as in the absence of radiation.

For intermediate quantum path lengths (η << ℓ << L), the x^2 domain being generated for x ~ ℓ expands as the radiation grows. The "2/3" law holds outside the square section, where there is no influence of radiation in the domain of large scales and the fluctuation level is reduced strongly for small x as compared with the case of no radiation because of the smoothing action of radiation in the larger vortices of dimension ~ℓ.

For ℓ >> L, radiation influences the form of the structural function in the whole range of scales. For values of the argument of the order of the external scale, the form of the structural function is determined by the pumping mode. In the inertial interval in the domain of small arguments where radiation decay of the inhomogeneities proceeds more slowly than the hydrodynamic, the "2/3" law is valid. As radiation grows, the size of the domain where the "2/3" law holds is reduced. The fluctuation amplitude is here lowered simultaneously because of the smoothing action of radiation in large-scale inhomogeneities.

In conclusion, we present an estimate of the conditions under which radiation starts to influence the form of the structural function. Combining the conditions (15), (21), and (37), we obtain that the radiation effects become substantial for

$$\frac{5\cdot 10^{-5}\langle T\rangle^4}{Pv} \gtrsim \begin{cases} \eta l^{-1}\mathrm{Re}^{-1/4} \sim \dfrac{L}{l\,\mathrm{Re}}, & l \ll \eta, \\ \left(\dfrac{l}{L}\right)^{1/3}, & \eta \ll l \ll L, \\ \dfrac{l}{L}, & l \gg L, \end{cases} \tag{42}$$

where P is the gas pressure, atm; v is the fluctuating external scale velocity, m/sec; and Re is the Reynolds number of the external scale of turbulence. The value of the specific heat corresponding to a diatomic gas with frozen vibrations was used in deducing these conditions. It is seen from (42) that the minimal temperature T_{min} for which the influence of radiation is possible is realized for $\ell = \eta$.

The flow parameters Re = 10^4, v = 5-50 m/sec, P = 1 atm can be taken as typical parameters. Hence, T_{min} ~ 560-990°K. At such temperatures smoothing of just the shallowest temperature inhomogeneities, of the order of the internal scale, is possible. The lowest temperature T_L at which radiation smoothes the temperature fluctuations of the external scale is realized for ℓ = L. We obtain from (42) for the parameters presented above for the medium: T_L ~ 990-1760 °K. The temperatures for which distortion of the structural function in the inertial interval is possible lie between those mentioned.

NOTATION

T, temperature; v, velocity of the medium; q, thermal source power; χ, thermal diffusivity coefficient; ρ, c_p, the density and specific heat of the medium at constant pressure; κ, absorption coefficient; ℓ, quantum path length; B_ω, Planck function at the frequency ω; W, function defined in (2); D, structural function of the temperature field; $R_{ij}(x)$, tensor defined in (4): R(x), longitudinal part of the tensor R_{ij}; σ, Stefan–Boltzmann constant; h, parameter defined in (5); N, power of the temperature inhomogeneity sources defined in (6); ν, kinematic viscosity; η, L, the internal and external turbulence scales; ε, rate of kinetic energy dissipation; P, pressure; Re, Reynolds number; τ_x, $\tau^r{}_x$, hydrodynamic and radiation decay times of the temperature inhomogeneity of dimension x; x_0, x_1, boundaries of strong influence of radiation on the form of the structural function; x, y, z, radius vectors; t, time; ', derivative with respect to the coordinate; <...>, average over the ensemble.

LITERATURE CITED

1. A. S. Monin and A. M. Yaglom, Statistical Fluid Mechanics, Vol. 2, MIT Press (1975).
2. A. M. Obukhov, "Structure of the temperature field in a turbulent flow," Izv. Akad. Nauk SSSR, Ser. Geogr. Geofiz., 13, No. 1, 58-96 (1949).
3. A. M. Yaglom, "On the local structure of the temperature field in a turbulent flow," Dokl. Akad. Nauk SSSR, 69, No. 6, 743-746 (1949).
4. V. M. Ievlev, Turbulent Motions of High-Temperature Continuous Media [in Russian], Nauka, Moscow (1975).
5. A. A. Townsend, "The effect of radiative transfer on turbulent flow of stratified fluid," J. Fluid Mech., 4, No. 4, 361-375 (1958).
6. G. M. Shved, "On the rate of radiant damping of turbulent temperature fluctuations in upper atmospheres of planets," Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 7, No. 11, 1132-1142 (1971).
7. G. M. Shved and R. A. Akmaev, "Influence of radiative heat transfer on turbulence in planetary atmospheres," Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 10, No. 8, 849-898 (1974).
8. V. I. Naidenov and S. A. Shindin, "On interaction between radiation and turbulent temperature fluctuations in the boundary layer," Teplofiz. Vys. Temp., 19, No. 1, 136-139 (1981).
9. E. A. Novikov, "Functionals and the method of random forces in the theory of turbulence," Zh. Eksp. Teor. Fiz., 47, No. 5, 1919-1926 (1964).

10. V. I. Klyatskin, Statistical Description of Dynamical Systems with Fluctuating Parameters [in Russian], Nauka, Moscow (1975).
11. V. I. Klyatskin, Stochastic Equations and Waves in Randomly Inhomogeneous Media [in Russian], Nauka, Moscow (1980).

MATHEMATICAL MODELING OF COMBINED HEAT TRANSFER IN DISPERSED MATERIALS

O. M. Alifanov, B. P. Gerasimov, T. G. Elizarova, V. K. Zantsev, B. N. Chetverushkin, and E. V. Shil'nikov

UDC 517.9:956.2/3

We show that combined heat transfer in a dispersed medium can be modeled numerically by treating convective and radiative–conductive heat transfer separately. We refine the radiative heat-transfer model by comparison with experiment.

In the develoment of a thermal shield of various technical system and aggregates of dispersed materials, it is necessary to study their thermal insulation properties in detail. The heat-transport mechanism in such materials is rather complicated, and includes heat transport directly as a result of the thermal conductivity of the material (conductive heat transport) and heat transport by radiation (radiative heat transport). If the porous medium is filled with gas, there may also be convective heat transfer. A purely experimental study of heat-transport processes and the resulting heat fluxes is difficult, and, therefore, a simultaneous study by full-scale and numerical experiments can give good results [1, 2].

In the present article we consider the methodical aspects of the mathematical modeling of combined heat transfer in a dispersed material based on optically transparent dispersed silicic materials with a 90% and more porosity of the sample used. The samples were rectangular parallelepipeds. It is required to determine the temperature of the lower surface of the sample for a specified time dependence of the temperature of its upper surface.

A general formulation of this problem includes the combined consideration of the system of equations describing radiative transport (taking account of absorption, emission, and scattering) and the laws of conservation of energy and momentum in the gas. The purpose of the study is to construct a mathematical model, to ascertain the role of each heat-transfer mechanism, to compare various methods of calculation, and to develop the optimal approach to the solution of the problem. As the most reasonable and technically relatively simply realizable approach we propose a procedure based on the separate treatment of convective and radiative–conductive heat transfer.

1. Investigation of Convective Heat Transport in a Porous Medium. We describe the nonlinear filtration of a liquid in a porous medium by the Navier–Stokes equations, which in the Boussinesq approximation, taking account of Darcy's law, we write in the following form [3]:

$$\frac{\partial \bar{\mathbf{V}}}{\partial t} + (\bar{\mathbf{V}}\nabla)\bar{\mathbf{V}} = -\frac{1}{\rho}\nabla P + \nu\Delta\bar{\mathbf{V}} + \beta\bar{g}T - K\bar{\mathbf{V}}, \tag{1}$$

$$\frac{\partial T}{\partial t} + (\bar{\mathbf{V}}\nabla)T = \chi\Delta T, \tag{2}$$

$$\nabla\bar{\mathbf{V}} = 0, \tag{3}$$

where $K = \nu\delta_{ij}/C_{\Phi i}$, $C_{\Phi i} = k_{\Phi i}\nu/g$ is the penetrability, which characterizes the geometrical properties of the porous medium, cm^2; the value of $C_{\Phi i}$ does not depend on the kind of filter-

M. V. Keldysh Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 5, pp. 781-791, November, 1985. Original article submitted December 21, 1984.

 0022-0841/85/4905-1306$09.50